

Supplementary Material

10.1302/2633-1462.44.BJO-2023-0024.R1



Fig. a. Geometric pattern to describe the angles and measurements used to demonstrate the calculated C'KS angle (the impact of the diaphyseal femoral deformity on the knee alignment).

The aim of this demonstration was to calculate the ε_2 angle and the β angle knowing the other elements.

I: Bone length between the knee and the end of the shaft (I = DK + DC').

HC': length of the femoral neck.

DC': Distance between the top of the femoral bowing and the top of the NSA angle (C').

DK: Distance between the knee centre (K) and the top of the femoral bowing (Cora = D)

 α = NSA: Angle of intersection between the femoral neck axis and the proximal femoral shaft axis.

 $\varphi = 180^{\circ} - FBow$

 β = C'KS: Angle between the distal femoral shaft axis and the line joining C' and the knee centre K.

 Ω = HKC': Angle between the mechanical axis line of the femur (between the hip centre and the knee centre) and the line joining C' and the knee centre K.

 ε = aMDFA: Medial angle formed between the distal femoral shaft axis and the knee joint line of the femur in the frontal plane.

 ε_2 = mMDFA: Medial angle formed between the mechanical axis line of the femur (HK) and the knee joint line of the femur in the frontal plane.

Step 1: Calculation of the distance KC'

If we use the AI-Kashi theorem on the triangle C'DK in order to obtain the distance KC', we have:

$$KC'^{2} = DK^{2} + DC'^{2} - 2 \times DK \times DC' \times \cos\varphi$$

$$KC'^2 = DK^2 + DC'^2 - 2 \times l \times DC' \times \cos \varphi$$

$$KC' = \sqrt{(2DK^2 - 2 \times l \times DK)(1 + \cos \varphi) + l^2}$$

Step 2: Calculation of the β angle

Using the Al-Kashi theorem in the same triangle as before, we have:

$$DC'^2 = DK^2 + KC'^2 - 2 \times DK \times KC' \times \cos\beta$$

$$\cos \beta = \frac{DK^2 + KC'^2 - DC'^2}{2 \times DK \times KC'}$$
$$\cos \beta = \frac{DK^2 + (2DK^2 - 2 \times l \times DK)(1 + \cos \varphi) + l^2 - (l - DK)^2}{2 \times DK \times \sqrt{(2DK^2 - 2 \times l \times DK)(1 + \cos \varphi) + l^2}}$$

$$\cos \beta = \frac{2DK^2 + (2DK^2 - 2 \times l \times DK)(1 + \cos \varphi) - 2 \times l \times DK}{2 \times DK \times \sqrt{(2DK^2 - 2 \times l \times DK)(1 + \cos \varphi) + l^2}}$$
$$\cos \beta = \frac{(2DK^2 - 2 \times l \times DK)(2 + \cos \varphi)}{2 \times DK \times \sqrt{(2DK^2 - 2 \times l \times DK)(1 + \cos \varphi) + l^2}}$$

$$\cos \beta = \frac{2 \times DK \times (DK - l)(2 + \cos \varphi)}{2 \times DK \times \sqrt{(2DK^2 - 2 \times l \times DK)(1 + \cos \varphi) + l^2}}$$
$$\cos \beta = \frac{(DK - l)(2 + \cos \varphi)}{\sqrt{(2DK^2 - 2 \times l \times DK)(1 + \cos \varphi) + l^2}}$$

Step 3: Angle θ

$$180 = \theta + \varphi + \beta$$
$$\theta = 180 - \varphi - \beta$$

Step 4: Distance KH

Using the Al-Kashi theorem in the triangle C'HK, we have:

$$KH^{2} = KC'^{2} + HC'^{2} - 2 \times KC' \times HC' \times \cos(\alpha - \theta)$$

$$KH^{2} = KC'^{2} + HC'^{2} - 2 \times KC' \times HC' \times \cos(\alpha - \theta)$$

Step 5: Calculation of the angle Ω

Using the Al-Kashi theorem in the triangle C'HK, we have:

$$HC'^{2} = KC'^{2} + KH^{2} - 2 \times KC' \times \cos \Omega$$
$$\cos \Omega = \frac{KC'^{2} + KH^{2} - HC'^{2}}{2 \times KC' \times KH}$$

Step 6: Calculation of the ϵ_2 angle

$$\varepsilon = \varepsilon_2 + \Omega + \beta$$
$$\varepsilon_2 = \varepsilon - \Omega - \beta$$